Homework [Ex-22(VI) 1-5]

DATE
PAGE No.

Ex-22(VI)

2. (a) Expand;

i) $(x+2)^3$

→ $(x)^3 + 3(x)^2 \cdot 2 + 3 \cdot x \cdot (2)^2 + (2)^3$

→ $x^3 + 6x^2 + 12x + 8$

ii) $(y-4)^3$

→ $(y)^3 - 3 \cdot (y)^2 \cdot 4 + 3 \cdot y \cdot (4)^2 - (4)^3$

→ $y^3 - 12y^2 + 48y - 64$

iii) $(2a+1)^3$

→ $(2a)^3 + 3 \cdot (2a)^2 \cdot 1 + 3 \cdot 2a \cdot (1)^2 + (1)^3$

→ $8a^3 + 12a^2 + 6a + 1$

iv) $(a-2b)^3$

→ $(a)^3 - 3 \cdot (a)^2 \cdot 2b + 3 \cdot a \cdot (2b)^2 - (2b)^3$

→ $a^3 - 6a^2b + 12ab^2 - 8b^3$

b) Find the cube of;

i) $z+5$

$= (z+5)^3$

$= (z)^3 + 3\cdot(z)^2\cdot 5 + 3\cdot z\cdot(5)^2 + (5)^3$

$= z^3 + 15z^2 + 75z + 125$

ii) $p-1$

$= (p-1)^3$

$= (p)^3 - 3\cdot(p)^2\cdot 1 + 3\cdot p\cdot(1)^2 - (1)^3$

$= p^3 - 3p^2 + 3p - 1$

iii) $3x-2$

$= (3x-2)^3$

$= (3x)^3 - 3\cdot(3x)^2\cdot 2 + 3\cdot 3x\cdot(2)^2 - (2)^3$

$= 27x^3 - 54x^2 + 36x - 8$

iv) $3-2a$

$= (3-2a)^3$

$\Rightarrow (3)^3 - 3\cdot(3)^2\cdot 2a + 3\cdot 3\cdot(2a)^2 - (2a)^3$

$= 27 - 54a + 36a^2 - 8a^3$

v) $4a+3b$

$= (4a+3b)^3$

$= (4a)^3 + 3\cdot(4a)^2\cdot 3b + 3\cdot 4a\cdot(3b)^2 + (3b)^3$

$= 64a^3 + 144a^2b + 108ab^2 + 27b^3$

vi) $2x-5y$

$= (2x-5y)^3$

$= (2x)^3 - 3\cdot(2x)^2\cdot 5y + 3\cdot 2x\cdot(5y)^2 - (5y)^3$

$= 8x^3 - 60x^2y + 150xy^2 - 125y^3$

vii) a^2+bc

$= (a^2+bc)^3$

$= (a^2)^3 + 3\cdot(a^2)^2\cdot bc + 3\cdot a^2\cdot(bc)^2 + (bc)^3$

$= a^6 + 3a^4bc + 3a^2b^2c^2 + b^3c^3$

viii) x^2+y^2

$= (x^2+y^2)^3$

$= (x^2)^3 + 3\cdot(x^2)^2\cdot y^2 + 3\cdot x^2\cdot(y^2)^2 + (y^2)^3$

$= x^6 + 3x^4y^2 + 3x^2y^4 + y^6$

2. Express the following expressions in $(a+b)^3$ or $(a-b)^3$ form;

a) $8x^3+12x^2y+6xy^2+y^3$

$= (2x)^3+3x4x^2y+3x2xy^2+(y)^3$

$= (2x)^3+3\times(2x)^2\times y+3\times 2x\times(y)^2+(y)^3$

$= (2x+y)^3$

---

b) $a^3-9a^2b+27ab^2-27b^3$

$= (a)^3-3x3a^2b+3x9ab^2-(3b)^3$

$= (a)^3-3\times(a)^2\times 3b+3\times a\times(3b)^2-(3b)^3$

$= (a-3b)^3$

c) $8x^3+36x^2y+54xy^2+27y^3$

$= (2x)^3+3x12x^2y+3x18xy^2+(3y)^3$

$= (2x)^3+3\times(2x)^2\cdot 3y+3\times 2x\times(3y)^2+(3y)^3$

$= (2x+3y)^3$

d) $27p^3-54p^2q+36pq^2-8q^3$

$= (3p)^3-3x18p^2q+3x12pq^2-(2q)^3$

$= (3p)^3-3\times(3p)^2\times 2q+3\times 3p\times(2q)^2-(2q)^3$

$= (3p-2q)^3$

e) $64x^3-240x^2y+300xy^2-125y^3$

$= (4x)^3 - 3\times80x^2y + 3\times100xy^2 - (5y)^3$

$= (4x)^3 - 3\times(4x)^2\times5y + 3\times4x\times(5y)^2 - (5y)^3$

$= (4x-5y)^3$

3) Factorize;

a) $1+x^3$

$= (1)^3 + (x)^3$

$= (1+x)(1^2 - 1\times x + (x)^2)$ [$\because a^3+b^3 = (a+b)(a^2-ab+b^2)$]

$= (1+x)(1-x+x^2)$

b) $y^3 - 27$

$= (y)^3 - (3)^3$

$= (y-3)((y)^2 + y\times3 + (3)^2)$ [$\because a^3-b^3 = (a-b)(a^2+ab+b^2)$]

$= (y-3)(y^2+3y+9)$

c) $8x^3+1$

$= (2x)^3 + (1)^3$

$= (2x+1)((2x)^2 - 2x\times1 + (1)^2)$

$= (2x+1)(4x^2-2x+1)$

d) $27p^3 - 8q^3$

$= (3p)^3 - (2q)^3$

$= (3p-2q)((3p)^2 + 3p \times 2q + (2q)^2)$

$= (3p-2q)(9p^2 + 6pq + 4q^2)$

e) $125a^3 + 27b^3$

$= (5a)^3 + (3b)^3$

$= (5a+3b)((5a)^2 - 5a \times 3b + (3b)^2)$

$= (5a+3b)(25a^2 - 15ab + 9b^2)$

f) $216 - p^3$

$= (6)^3 - (p)^3$

$= (6-p)((6)^2 + 6 \times p + (p)^2)$

$= (6-p)(36 + 6p + p^2)$

4) Simplify;

DATE
PAGE No.

a) $a^3+b^3-(a+b)^3$

$= (a+b)^3 - 3ab(a+b) - (a+b)^3$

$= -3ab(a+b)$

b) $(p+q)^3-(p-q)^3$

$= (p)^3+3p^2q+3pq^2+(q)^3-[(p)^3-(q)^3-3p^2q+3pq^2]$

$= p^3+3p^2q+3pq^2+q^3-p^3+q^3+3p^2q-3pq^2$

$= 3p^2q+3p^2q+q^3+q^3$

$= 6p^2q+2q^3$

$= 2q(3p^2+q)$

4(c) $(x+2y)^3+(x-2y)^3$ [Simplify]

-> Soln,

Here,

We know: $a^3+b^3=(a+b)(a^2-ab+b^2)$

Now,

$= (x+2y)^3+(x-2y)^3$

$= (x+2y+x-2y)\{(x+2y)^2-(x+2y)(x-2y)+(x-2y)^2\}$

$= 2x\{x^2+2\cdot x\cdot 2y+(2y)^2-\{x^2-(2y)^2\}+x^2-2\cdot x\cdot 2y+(2y)^2$

$= 2x\{x^2+4xy+4y^2-x^2+4y^2+x^2-4xy+4y^2\}$

$= 2x(x^2+12y^2)$

d) $(x-y)^3 + 3xy(x-y) \rightarrow (a-b)^3 + 3ab(a-b)$

$= x^3 - y^3 \rightarrow a^3 - b^3$

4(e) $27m^3+125n^3-(3m+5n)^3$

→ Soln,

Here,

$= 27m^3+125n^3-(3m+5n)^3$

$= 27m^3+125n^3-\{(3m)^3+3(3m)^2.5n+3.3m.(5n)^2+(5n)^3\}$

DATE
PAGE No.

$$= 27m^3 + 125n^3 - 27m^3 - 135m^2n - 225mn^2 - 125n^3$$

$$= -135m^2n - 225mn^2$$

$$= -45mn(3m + 5n)$$

6=2
9=3

5 a) If $a+b=10$ and $ab=16$, find a^3+b^3.

[∵ Note: Donot use factorise formula in this question.]

→ Soln

Here,

$a+b=10$

$ab=16$

Now,

$(a^3+b3) = (a+b)^3 - 3ab(a+b)$

$= (10)^3 - 3\times16(10)$

$= 1000 - 3\times160$

$= 1000 - 480$

$\therefore a^3+b^3 = 520$

u(e) redo

→

5(b) If $m+n=6$ and $mn=8$, find m^3+n^3.

→ Soln

Here,

$m+n=6$

$mn=8$

Now,

$m^3+n^3 = (m+n)^3 - 3mn(m+n)$

$= (6)^3 - 3\times 8(6)$

$= 216 - 3\times 48$

$= 216 - 144$

$\therefore m^3+n^3 = 72$

5(c) If $x-y=7$ and $xy=30$, find x^3-y^3.

→ Soln

Here,

$x-y=7$

$xy=30$

Now,

$x^3-y^3 = (x-y)^3 + 3xy(x-y)$

$= (7)^3 + 3\times 30(7)$

$= 343 + 90\times 7$

$= 3437630$
$= 973$

$\therefore x^3 - y^3 = 973$

d) If $p-q=8$ and $pq=20$, find p^3-q^3

→ Soln,

Here,

$p-q=8$

$pq=20$

Now,

$p^3-q^3 = (p-q)^3 + 3pq(p-q)$

$= (8)^3 + 3 \times 20 \times 8$

$= 512 + 60 \times 8$

$= 512 + 480$

$= 992$

$\therefore p^3-q^3 = 992$

e) If $x+\frac{1}{x}=3$, find $x^3+\frac{1}{x^3}$.

→ Soln

Here,

$$x+\frac{1}{x}=3$$

Now,

~~$x^3+\frac{1}{x^3}=$~~

Cubing both sides,

$$\left(x+\frac{1}{x}\right)^3=(3)^3$$

~~or, $(x)^3+\left(\frac{1}{x}\right)^3+3(x)^2\frac{1}{x}+3\cdot x\left(\frac{1}{x}\right)^2=9$~~

~~or, $x^3+\frac{1}{x^3}+3x+\frac{3}{x}=9$~~

or, $(x)^3+\left(\frac{1}{x}\right)^3+3\cdot x\cdot\frac{1}{x}\left(x+\frac{1}{x}\right)=27$

or, $x^3+\frac{1}{x^3}+3\times(3)=27$

or, $x^3+\frac{1}{x^3}+9=27$

or, $x^3 - \frac{1}{x^3} = 27 - 9$

$\therefore x^3 - \frac{1}{x^3} = 18$

7) If $y + \frac{1}{y} = a$, show that $y^3 + \frac{1}{y^3} = a^3 - 3a$

→ Soln,

Here,

$$y + \frac{1}{y} = a$$

Now,

Cubing both sides,

$$\left(y + \frac{1}{y}\right)^3 = (a)^3$$

or, $(y)^3 + \left(\frac{1}{y}\right)^3 + 3 \cdot y \cdot \frac{1}{y}\left[y + \frac{1}{y}\right] = a^3$

or, $y^3 + \frac{1}{y^3} + 3 \times a = a^3$

u(c) redo →

DATE
PAGE No.

or, $y^3 + \frac{1}{y^3} + 3a = a^3$

$\therefore y^3 + \frac{1}{y^3} = a^3 - 3a$

Hence Shown

g) If $z - \frac{1}{z} = 4$, find $z^3 - \frac{1}{z^3}$.

→ Soln,

Here,

$$z - \frac{1}{z} = 4$$

Now, [Cubing both sides]

$$\left(z^3 - \frac{1}{z^3}\right) = (4)^3$$

or, $(z)^3 - \left(\frac{1}{z}\right)^3 - 3 \cdot z \cdot \frac{1}{z}\left(z - \frac{1}{z}\right) = 64$

or, $z^3 - \frac{1}{z^3} - 3 \times 4 = 64$

or, $z^3 - \frac{1}{z^3} - 12 = 64$

or, $z^3 - \frac{1}{z^3} = 64 + 12$

$\therefore z^3 - \frac{1}{z^3} = 76$

h) If $a - \frac{1}{a} = p$, show that $a^3 - \frac{1}{a^3} = p^3 + 3p$.

→ Soln,

Here,

$$a - \frac{1}{a} = p$$

Now,

Cubing both sides,

$$\left(a - \frac{1}{a}\right)^3 = (p)^3$$

or, $(a)^3 - \left(\frac{1}{a}\right)^3 - 3 \cdot a \cdot \frac{1}{a}\left[a + \frac{1}{a}\right] = p^3$

or, $a^3 - \frac{1}{a^3} - 3 \times p = p^3$

$\therefore a^3 - \frac{1}{a^3} = p^3 + 3p$

Hence shown