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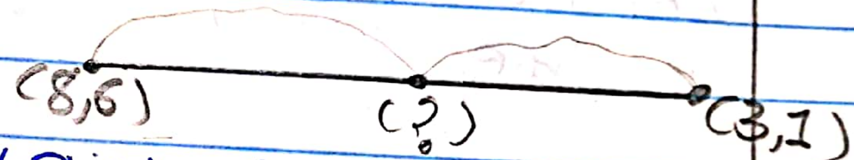
Exercise 3(II)

1) Find the coordinates of a point dividing internally the line joining the points;

i) (8,6) and (3,1) in the ratio of 3:2

→ Sol^n:

Here,



$(8,6) = (x_1, y_1)$

$(3,1) = (x_2, y_2)$

$m_1 : m_2 = 3:2$

Let the required point be (x,y).

We have,

① $x = \dfrac{m_1x_2 + m_2x_1}{m_1 + m_2} = \dfrac{3\times3+2\times8}{3+2}$

$= \dfrac{9+16}{5} = \dfrac{25}{5} = 5$

⑪ $y = \dfrac{m_1y_2 + m_2y_1}{m_1+m_2} = \dfrac{3\times1+2\times6}{3+2}$

$= \dfrac{3+12}{5} = \dfrac{15}{5} = 3$

∴ The required point $= (x,y) = (5,3)$

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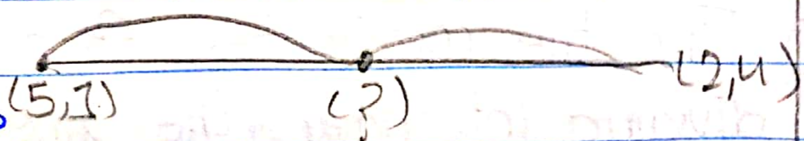
b) (5,1) and (2,4) in the ratio 2:1.

→ Soln,



Here,

$$(5,1) = (x_1, y_1)$$
$$(2,4) = (x_2, y_2)$$
$$m_1 : m_2 = 2:1$$

Let the required point be (x,y)

We have,

$$(x,y) = \left(\frac{m_1x_2 + m_2x_1}{m_1+m_2}, \frac{m_1y_2 + m_2y_1}{m_1+m_2}\right)$$
$$= \left(\frac{2\times2+1\times5}{2+1}, \frac{2\times4+1\times1}{2+1}\right)$$
$$= \left(\frac{4+5}{3}, \frac{8+1}{3}\right)$$
$$= \left(\frac{9}{3}, \frac{9}{3}\right)$$
$$= (3,3)$$

∴ The required point is $(x,y) = (3,3)$.

---

c) (6,−5) and (−1,2) in the ratio of 3:2.

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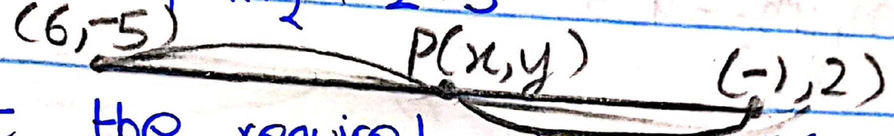
→ Soln,

Here,

Let the points $\overset{x_1\ y_1}{(6,-5)}$ & $\overset{x_2\ y_2}{(-1,2)}$ be A and B respectively.

$m_1 : m_2 = 2:5$



Let the required point be ~~P(~~ P(x,y).

We have,

$$P(x,y) = \left(\frac{m_1x_2 + m_2x_1}{m_1+m_2}, \frac{m_1y_2+m_2y_1}{m_1+m_2}\right)$$

$$= \left(\frac{2\times -1 + 5\times 6}{2+5}, \frac{2\times 2 + 5\times -5}{2+5}\right)$$

$$= \left(\frac{-2+30}{7}, \frac{4+(-25)}{7}\right)$$

$$= \left(\frac{28}{7}, \frac{-21}{7}\right)$$

$$= (4,-3)$$

∴ Reqd. point = P(4,-3)

---

d) (-10, 12) and (-3,-9) in the ratio 4:3

→) Sol^n:

Here,

Let the points be A(-10,12) and B(-3,-9). Let the required point be P(x,y).

$m_1 : m_2 = 4:3$

We have,

$$P(x,y) = \left(\frac{m_1x_2+m_2x_1}{m_1+m_2}, \frac{m_1y_2+m_2y_1}{m_1+m_2}\right)$$

$$= \left(\frac{4\times-3+3\times-10}{4+3}, \frac{4\times-9+3\times12}{4+3}\right)$$

$$= \left(\frac{-12+(-30)}{7}, \frac{-\cancel{36}+\cancel{36}}{7}\right)$$

$$= \left(\frac{-42}{7}, \frac{0}{7}\right)$$

$$= (-6, 0)$$

$\therefore$ Reqd. point is P(-6,0).

2. Find the coordinates of a point dividing externally the line joining

the points:

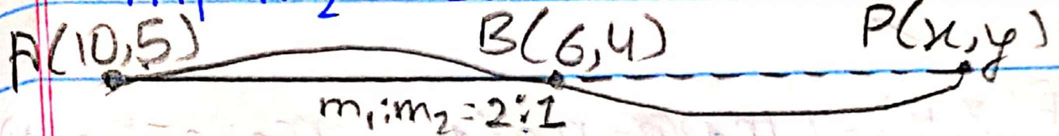
a) (10,5) and (6,4) in the ratio 2:1.

→ Soln,

Here,

Let the points be $A(10,5)$ (x_1, y_1) and $B(6,4)$ (x_2, y_2). Let the reqd. point be $P(x,y)$.

$m_1 : m_2 = 2:1$



We have,

$$P(x,y) = \left(\frac{m_1x_2 - m_2x_1}{m_1 - m_2}, \frac{m_1y_2 - m_2y_1}{m_1 - m_2} \right)$$

$$= \left(\frac{2\times6 - 1\times10}{2-1}, \frac{2\times4 - 1\times5}{2-1} \right)$$

$$= \left(\frac{12-10}{1}, \frac{8-5}{1} \right)$$

$$= (2,3)$$

∴ Reqd. point is $P(2,3)$

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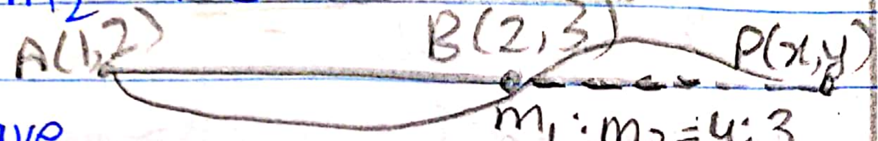
b) (1,2) and (2,3) in the ratio 4:3

→ Sol^n,

Here,

Let the required point be $P(x,y)$ and given points be $A(1,2)$ and $B(2,3)$.

$m_1 : m_2 = 4:3$



We have,

$$P(x,y) = \left(\frac{m_1x_2 - m_2x_1}{m_1 - m_2}, \frac{m_1y_2 - m_2y_1}{m_1 - m_2}\right)$$

$$= \left(\frac{\cancel{3\times}4\times2 - 3\times1}{4-3}, \frac{4\times3 - 3\times2}{4-3}\right)$$

$$= \left(\frac{8-3}{1}, \frac{12-6}{1}\right)$$

$$= (5,6)$$

∴ Reqd. point is $P(5,6)$.

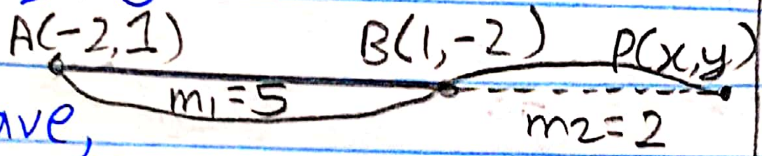
c) (−2,1) and (1,−2) in the ratio ~~3~~ 5:2.

→ Soln,

Here,

Let the points be A(-2,1) & B(1,-2) and ~~p~~required point be P(x,y).

$m_1 : m_2 = 5:2$



We have,

$$P(x,y) = \left(\frac{m_1x_2 - m_2x_1}{m_1 - m_2}, \frac{m_1y_2 - m_2y_1}{m_1 - m_2}\right)$$

$$= \left(\frac{5\times1 - 2\times-2}{5-2}, \frac{5\times-2 - 2\times1}{5-2}\right)$$

$$= \left(\frac{5+4}{3}, \frac{-10-2}{3}\right)$$

$$= \left(\frac{9}{3}, \frac{-12}{3}\right)$$

$$= (3, -4)$$

∴ Reqd. point is P(3,-4)

---

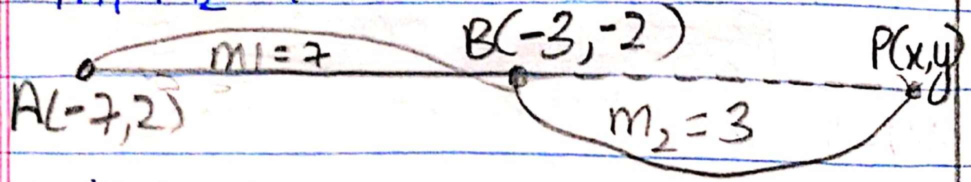
d) (-7,2) and (-3,-2) in the ratio of 4:3.

→ Soln,

Here,

Let the points be $A(-7,2)$ and $B(-3,-2)$ and required point be $P(x,y)$.

$m_1 : m_2 = 7:3$.



We have,

$$P(x,y) = \left(\frac{m_1x_2 - m_2x_1}{m_1 - m_2}, \frac{m_1y_2 - m_2y_1}{m_1 - m_2}\right)$$

$$= \left(\frac{7\times -3 - 3\times -7}{7-3}, \frac{7\times -2 - 3\times 2}{7-3}\right)$$

$$= \left(\frac{-\cancel{21}+\cancel{21}}{4}, \frac{-14-6}{4}\right)$$

$$= \left(\frac{0}{4}, \frac{-\cancel{20}^{5}}{\cancel{4}_{1}}\right)$$

$$= (0,-5)$$

∴ Reqd. point is $P(0,-5)$.

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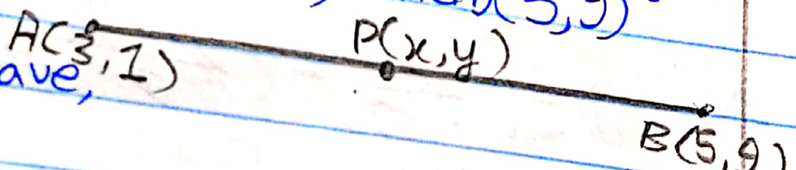
3. Find the coordinates of the middle point of the line joining the points;

i) (3,1) and (5,9)

→ Soln

Here,

Let the mid point be $P(x,y)$ and points be $A(3,1)$ and $B(5,9)$



We have,

$$P(x,y) = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$= \left(\frac{3+5}{2}, \frac{1+9}{2}\right)$$

$$= \left(\frac{8}{2}, \frac{10}{2}\right)$$

$$= (4,5)$$

$\therefore$ Mid-point $= P(4,5)$

---

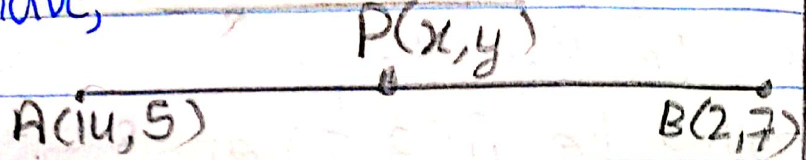
ii) (14,5) and (2,7)

→ Soln,

Here,

Let the points be $A(14,5)$ and $B(2,7)$ and mid-point be $P(x,y)$.

We have,



$$P(x,y) = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$= \left(\frac{14+2}{2}, \frac{5+7}{2}\right)$$

$$= \left(\frac{16}{2}, \frac{12}{2}\right)$$

$$= (8,6)$$

$\therefore$ Mid-point $= P(8,6)$

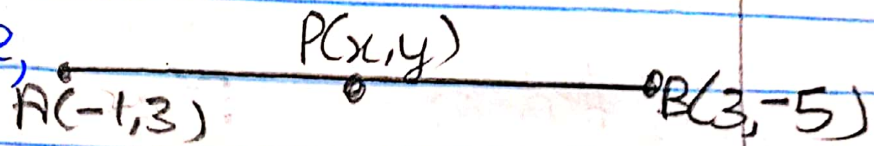
iii) $(-1,3)$ and $(3,-5)$

→ Soln,

Here,

Let the points be $A(-1,3)$ and $B(3,-5)$ and mid-point be $P(x,y)$

we have,



$$P(x,y) = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$= \left(\frac{-1+3}{2}, \frac{3+(-5)}{2}\right)$$

$$= \left(\frac{\cancel{2}}{\cancel{2}}, \frac{3-5}{2}\right)$$

$$= \left(1, \frac{\cancel{-2}}{\cancel{2}} -1\right)$$

$$= (1,-1)$$

$\therefore$ Mid-point $= P(1,-1)$

iv) $(0,-6)$ and $(-4,10)$

→ Soln,

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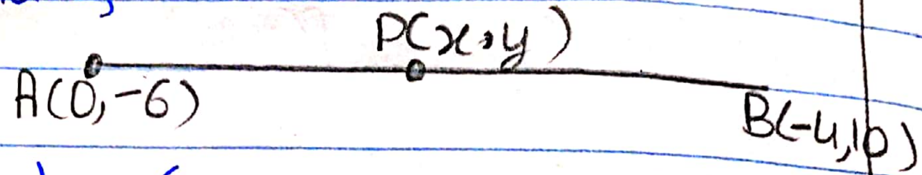
Here,
let the points be A(0,-6) and B(-4,10) and mid-point be P(x,y)

We have,



$$P(x,y) = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$= \left(\frac{0+(-4)}{2}, \frac{-6+10}{2}\right)$$

$$= \left(\frac{-4}{2}, \frac{4}{2}\right)$$

$$= (-2, 2)$$

$\therefore$ Mid-point = P(-2,2)

4) i) The middle point of line is (3,5) and one end of the line is (4,7). Find the coordinates ~~at~~ of the other end.

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Soln,

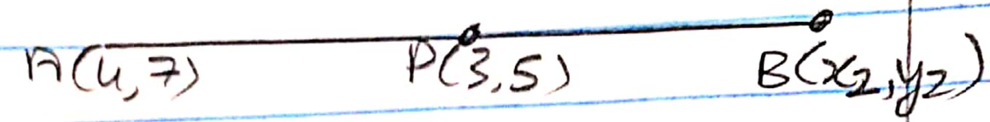
Here,

~~Let the points at be~~

Point at one end = $A(4,7)$

Mid Point = $P(3,5)$

Point at other end = ? or $B(x_2, y_2)$



We have,

$$P(3,5) = \left(\frac{4+x_2}{2}, \frac{7+y_2}{2}\right)$$

From the above condition,

Taking x component,

$$3 = \frac{4+x_2}{2}$$

or, $4+x_2 = 6$

or, $x_2 = 6-4 \quad \therefore x_2 = 2$

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Taking y component,

$$5 = \frac{7+y_2}{2}$$

or, $7+y_2 = 10$

or, $y_2 = 10-7$

$\therefore y_2 = 3$

$\therefore$ The point of other end = B(2,3)

ii) The middle point of a line is (0,-1) and one end of the line is (-3,5). ~~and~~ Find the coordinates of the other end.

→ Soln

Here,

Mid point = P(0,-1)

Point at one end = A(-3,5)

Point at another end = B(x_2,y_2)

We have,

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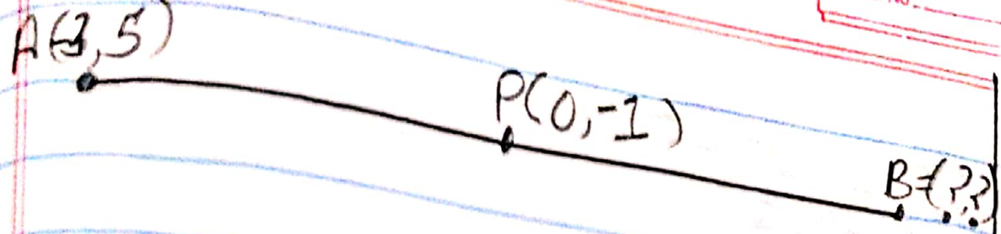


$$P(0,-1) = \left(\frac{-3+x_2}{2}, \frac{5+y_2}{2} \right)$$

From the above condition,

Taking x component,

$$0 = \frac{-3+x_2}{2}$$

or, $0 \times 2 = -3 + x_2$

or, $x_2 = 0 + 3$

$\therefore x_2 = 3$

Taking y component,

$$-1 = \frac{5+y_2}{2}$$

or, $5 + y_2 = 2 \times -1$

or, $y^2 = -2-5$

∴ $y^2 = -7$

∴ The point of another end = B(3,-7)

5) In what ratio does the line joining the points:

i) (3,1) and (8,6) divide by the point (5,3)?

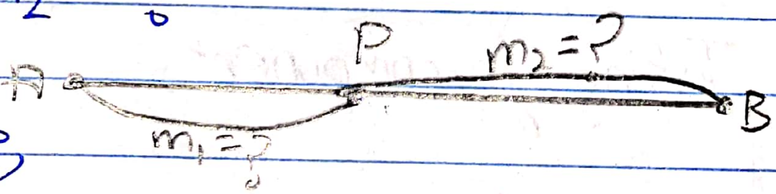
→ Soln,

Here,

Let the ~~po~~ points be A(3,1), B(8,6) and P(5,3).

$m_1 : m_2 = ?$

We have,



$$P(5,3) = \left(\cancel{m_1 \times 8 + m} \frac{m_1 \times 8 + m_2 \times 3}{m_1 + m_2} \right.,$$

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$$\frac{m_1 \times 6 + m_2 \times 1}{m_1 + m_2}\Big)$$

or, $P(5,3) = \left(\frac{8m_1 + 3m_2}{m_1 + m_2}, \frac{6m_1 + m_2}{m_1 + m_2}\right)$

From the above condition,
Taking x component,

$$5 = \frac{8m_1 + 3m_2}{m_1 + m_2}$$

or, $5(m_1 + m_2) = 8m_1 + 3m_2$

or, $5m_1 + 5m_2 = 8m_1 + 3m_2$

or, $5m_2 - 3m_2 = 8m_1 - 5m_1$

or, $2m_2 = 3m_1$

or, $\frac{m_2}{m_1} = \frac{3}{2}$

$\therefore$ $m_1 : m_2 = 2:3$

Ans.

ii) (5,-2) and (-2,5) divide by the point (2,1)?

→ Soln

Here,

Let the points be A(5,-2), B(-2,5) and P(2,1).

$m_1 : m_2 = ?$

We have,

$$P(2,1) = \left(\frac{m_1 \times -2 + m_2 \times 5}{m_1 + m_2}, \frac{m_1 \times 5 + m_2 \times -2}{m_1 + m_2}\right)$$

or, $$P(2,1) = \left(\frac{-2m_1 + 5m_2}{m_1 + m_2}, \frac{5m_1 - 2m_2}{m_1 + m_2}\right)$$

From the above condition,

Taking Y component,

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$$1 = \frac{5m_1 - 2m_2}{m_1 + m_2}$$

or, $m_1 + m_2 = 5m_1 - 2m_2$

or, $m_2 + 2m_2 = 5m_1 - m_1$

or, $3m_2 = 4m_1$

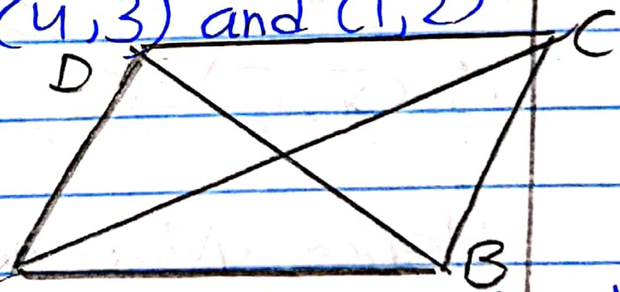
or, $\frac{3}{4} = \frac{m_1}{m_2}$

$\therefore$ $m_1 : m_2 = 3:4$

Ans.

6) Show that the following points represent the vertices of a parallelogram;

i) $(-2,-1)$, $(1,0)$, $(4,3)$ and $(1,2)$



→ soln,

Here,

Let $A(-2,-1)$, $B(1,0)$, $C(4,3)$ and $D(1,2)$ be 4 points

- DIAGONAL AC;

Let (x,y) be the mid-point of AC:

$$x = \frac{x_1+x_2}{2} = \frac{-2+4}{2} = \frac{\cancel{2}}{\cancel{2}} = 1$$

$$y = \frac{y_1+y_2}{2} = \frac{-1+3}{2} = \frac{\cancel{2}}{\cancel{2}} = 1$$

~~Mid.~~ $\therefore (x,y) = (1,1)$

- Diagonal BD;

Let $(\bar{x}, \bar{y})$ be midpoint of BD.

$$\bar{x} = \frac{x_1+x_2}{2} = \frac{1+1}{2} = \frac{2}{2} = 1$$

$$\bar{y} = \frac{y_1+y_2}{2} = \frac{0+2}{2} = \frac{2}{2} = 1$$

$\therefore (\bar{x}, \bar{y}) = (1,1)$

Hence, Mid-point of AC = mid point BD.

Or, $(x,y) = (\bar{x}, \bar{y}) = (1,1)$

ABCD are the vertices of a parallelogram.

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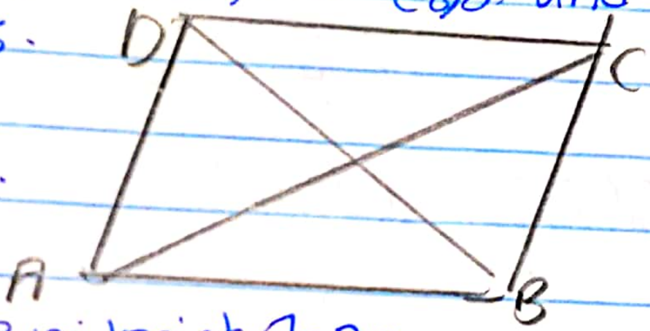
ii) (0,2), (2,5), (8,8) and (6,5)

→ Soln

Here,

Let A(0,2) B(2,5), C(8,8) and D(6,5) be 4 points.



- Diagonal AC.

Let (x,y) be midpoint of AC:

$$\cancel{x = x_1}(x,y) = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$= \left(\frac{0+8}{2}, \frac{2+8}{2}\right)$$

$$= \left(\frac{8}{2}, \frac{10}{2}\right)$$

$\therefore (x,y) = (4,5)$

- Diagonal BD,

Let $(\bar{x},\bar{y})$ be mid point of BD.

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$$(\bar{x}, \bar{y}) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$= \left(\frac{2+6}{2}, \frac{5+5}{2}\right)$$

$$= \left(\frac{8}{2}, \frac{10}{2}\right)$$

$\therefore (\bar{x}, \bar{y}) = (4, 5)$

Hence, $(x, y) = (\bar{x}, \bar{y}) = (4, 5)$

$\therefore$ ABCD are vertices of a parallelogram.

Proved

iii) $(4,5)$, $(9,7)$, $(7,3)$ and $(2,1)$

$\rightarrow$ Soln,

Here,

Let $A(4,5)$, $B(9,7)$, $C(7,3)$ and $D(2,1)$ be 4 points.

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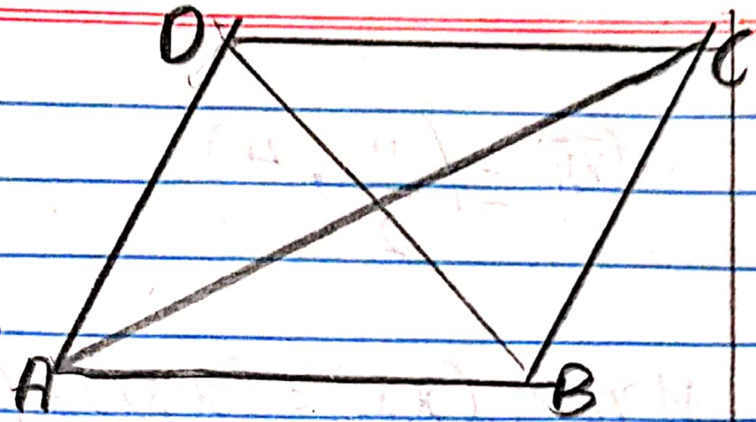

We have,

- diagonal AC;

Let (x,y) be midpoint of AC.

$$(x,y) = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$= \left(\frac{4+7}{2}, \frac{5+3}{2}\right)$$

$$= \left(\frac{11}{2}, \frac{8}{2}\right)$$

$$\therefore (x,y) = \left(\frac{11}{2}, 4\right)$$

- diagonal BD;

Let $(\bar{x},\bar{y})$ be mid point of BD.

$$(\bar{x},\bar{y}) = \left(\frac{9+2}{2}, \frac{7+1}{2}\right)$$

$$= \left(\frac{11}{2}, \frac{8}{2}\right)$$

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$$\therefore (\bar{x}, \bar{y}) = \left(\frac{11}{2}, 4\right)$$

Hence, $(x, y) = (\bar{x}, \bar{y}) = \left(\frac{11}{2}, 4\right)$

$\therefore$ ABCD are the vertices of a parallelogram.

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