Ex-2(IV)

1) Prove that:

(i) $\sin\theta \cdot \cot\theta = \cos\theta$

$= \frac{\sin\theta \cdot \cos\theta}{\sin\theta}$

→ Soln

Here,

$L.H.S = \sin\theta \times \cot\theta$

$= \frac{\sin\theta \times \cos\theta}{\sin\theta}$

$= \cos\theta$

$= R.H.S$ proved

ii) $\tan\theta \cdot \operatorname{cosec}\theta = \sec\theta$

→ Soln,

Here,

$L.H.S = \tan\theta \times \operatorname{cosec}\theta$

$= \frac{\sin\theta}{\cos\theta} \times \frac{1}{\sin\theta}$

$= \frac{1}{\cos\theta}$

$= \sec\theta$

$=$ R.H.S proved

iii) $\cos\theta \times \operatorname{cosec}\theta = \cot\theta$

→ Soln,

Here,

$$\text{L.H.S} = \cos\theta \times \operatorname{cosec}\theta$$

$$= \cos\theta \times \frac{1}{\sin\theta}$$

$$= \frac{\cos\theta}{\sin\theta}$$

$$= \cot\theta \quad \text{[PROVED]}$$

(iv) $\sin\theta \times \sec\theta = \tan\theta$

→ Soln,

Here,

$$\text{L.H.S} = \sin\theta \times \frac{1}{\cos\theta}$$

$$= \frac{\sin\theta}{\cos\theta}$$

$$= \tan\theta$$

$=$ R.H.S [Proved]

v) $\sec\theta \times \sin\theta \times \cot\theta = 1$

→ Soln

Here,

$$L.H.S = \sec\theta \times \sin\theta \times \cot\theta$$

$$= \frac{1}{\cancel{\cos\theta}} \times \frac{\cancel{\sin\theta}}{1} \times \frac{\cancel{\cos\theta}}{\cancel{\sin\theta}}$$

$$= 1$$

$$= R.H.S \text{ proved}$$

vi) $\dfrac{\sin\theta \times \operatorname{cosec}\theta}{\sec\theta} = \cos\theta$

→ Soln

Here,

$$L.H.S. = \frac{\sin\theta \times \operatorname{cosec}\theta}{\sec\theta}$$

$$= \frac{\cancel{\sin\theta} \times \frac{1}{\cancel{\sin\theta}}}{\sec\theta}$$

$$= \frac{1}{\sec\theta}$$

$$= \cos\theta = R.H.S \text{ proved}$$

vii) $\frac{\tan\theta \times \cot\theta}{\sec\theta \times \operatorname{cosec}\theta} = \sin\theta \times \cos\theta$

→ Soln,

Here,

$$L.H.S = \frac{\tan\theta \times \cot\theta}{\sec\theta \times \operatorname{cosec}\theta}$$

$$= \frac{\frac{\sin\theta}{\cos\theta} \times \frac{\cos\theta}{\sin\theta}}{\sec\theta \times \operatorname{cosec}\theta}$$

$$= \frac{1}{\sec\theta \times \operatorname{cosec}\theta}$$

$$= \frac{1}{\sec\theta} \times \frac{1}{\operatorname{cosec}\theta}$$

$$= \cos\theta \times \sin\theta$$

$$= \sin\theta \times \cos\theta$$

= RHS proved

viii) $\frac{\sin\theta \times \sec\theta}{\operatorname{cosec}\theta \times \cos\theta} = \tan^2\theta$

→ Soln,

Here,

$$L.H.S = \frac{\sin\theta \times \sec\theta}{\operatorname{cosec}\theta \times \cos\theta}$$

$$= \frac{\sin\theta \times \frac{1}{\cos\theta}}{\frac{1}{\sin\theta} \times \cos\theta}$$

$$= \frac{\frac{\sin\theta}{\cos\theta}}{\frac{\cos\theta}{\sin\theta}}$$

$$= \frac{\sin\theta}{\cos\theta} \times \frac{\sin\theta}{\cos\theta}$$

$$= \tan^2\theta$$

$$= \text{R.H.S proved}$$

ix) $\text{cosec}\theta \times \sqrt{1 - \sin^2\theta} = \cot^2\theta$

→ Soln,

Here,

$$\text{L.H.S} = \text{cosec}\theta \times \sqrt{1 - \sin^2\theta}$$

$$= \text{cosec}\theta \times \cos\theta$$

$$= \frac{1}{\sin\theta} \times \cos\theta$$

$$= \frac{\cos\theta}{\sin\theta}$$

$$= \cot\theta \text{ [RHS proved]}$$

x) $\sec\theta \times \sqrt{1-\cos^2\theta} = \tan\theta$

→ Soln

Here,

$$\text{L.H.S} = \sec\theta \times \sqrt{1-\cos^2\theta}$$
$$= \sec\theta \times \sin\theta$$
$$= \frac{1}{\cos\theta} \times \sin\theta$$
$$= \frac{\sin\theta}{\cos\theta}$$
$$= \tan\theta$$
$$= \text{R.H.S proved}$$

2. Prove that:

(i) $\sin^2A - \sin^2B = \cos^2B - \cos^2A$

→ Soln,

Here,

$$\text{L.H.S} = \sin^2A - \sin^2B$$
$$= 1 - \sin^2A - (1 - \sin^2B)$$
$$= \cos^2A - \cos^2B$$
$$= \text{Proved}$$

$= 1 - \cos^2 A - (1 - \cos^2 B)$

$= 1 - \cos^2 A - 1 + \cos^2 B$

$= \cos^2 B - \cos^2 A$

= Proved

ii) $\sec^2 A - \operatorname{cosec}^2 A = \tan^2 A - \cot^2 A$

→ Soln,

Here,

L.H.S = $\sec^2 A - \operatorname{cosec}^2 A$

$= 1 + \tan^2 A - (1 + \cot^2 A)$

$= 1 + \tan^2 A - 1 - \cot^2 A$

$= \tan^2 A - \cot^2 A$

= Proved

iii) $\cos^2\theta - \cos^2\theta \times \sin^2\theta = \cos^4\theta$

→ Soln,

Here,

L.H.S = $\cos^2\theta - \cos^2\theta \sin^2\theta$

$= \cos^2\theta (\quad$

→

L.H.S = $\cos^2\theta - \cos^2\theta \sin^2\theta$

$= \cos^2\theta (1 - \sin^2\theta)$

$= \cos^2\theta \times \cos^2\theta$

$= \cos^4\theta$

iv) $\sin^2\theta + \sin^2\theta \times \cot^2\theta = 1$

→ Soln,

Here,

L.H.S = $\sin^2\theta + \sin^2\theta \times \frac{\cos^2\theta}{\sin^2\theta}$

$= \sin^2\theta + \cos^2\theta$

$= 1$

= proved

v) $\sin^2\theta + \cos^2\theta \times \tan^2\theta = 2\sin^2\theta$

→ Soln

Here,

L.H.S = $\sin^2\theta + \cos^2\theta \times \tan^2\theta$

$= \sin^2\theta + \cos^2\theta \times \frac{\sin^2\theta}{\cos^2\theta}$

$= \sin^2\theta + \sin^2\theta$

$= 2\sin^2\theta$

= Proved

vi) $\sin^2\theta \times \cot\theta + \cos^2\theta \times \tan^2\theta = 1$

→ Soln,

Here,

L.H.S $= \sin^2\theta \times \cot^2\theta + \cos^2\theta \times \tan^2\theta$

$= \frac{\sin^2\theta \times \cos^2\theta}{\sin^2\theta} + \frac{\cos^2\theta \times \sin^2\theta}{\cos^2\theta}$

$= \cos^2\theta + \sin^2\theta$

$= 1$

= R.H.S proved

vii) $\cos\theta(1 + \tan^2\theta) = \sec\theta$

→ Soln,

Here,

L.H.S $= \cos\theta(1 + \tan^2\theta)$

$= \cos\theta \times \sec^2\theta$

$= \frac{1}{\sec\theta} \times \sec^2\theta$

$= \sec\theta$

= proved

viii) $\sin\theta(1+\cot^2\theta) = \operatorname{cosec}\theta$

=> Soln

Here,

$$L.H.S = \sin\theta \times (1+\cot^2\theta)$$
$$= \sin\theta \times \operatorname{cosec}^2\theta$$
$$= \frac{1}{\operatorname{cosec}\theta} \times \operatorname{cosec}^2\theta$$
$$= \operatorname{cosec}\theta$$
$$= R.H.S \text{ proved}$$

ix) $(1+\cot^2\theta)(1-\sin^2\theta) = \cot^2\theta$

=> Soln,

Here,

$$L.H.S = (1+\cot^2\theta)(1-\sin^2\theta)$$
$$= \operatorname{cosec}^2\theta \times \cos^2\theta$$
$$= \frac{1}{\sin^2\theta} \times \cos^2\theta$$
$$= \frac{\cos^2\theta}{\sin^2\theta}$$
$$= \cot^2\theta$$
$$= R.H.S \text{ proved}$$

x) $(1-\cos^2 A)(1+\tan^2 A) = \tan^2\theta$

→ Soln,

Here,

$LHS = (1-\cos^2 A) \times (1+\tan^2\theta)$

$= \sin^2 A \times \sec^2 A$

$= \sin^2 A \times \frac{1}{\cos^2 A}$

$= \frac{\sin^2 A}{\cos^2 A}$

$= \tan^2\theta$

$= R.H.S$ proved

xi) $\sin A.\sqrt{1-\cos^2 A} + \cos A.\sqrt{1-\sin^2 A} = 1$

→ Soln,

Here,

$L.H.S = \sin A.\sqrt{1-\cos^2 A} + \cos A.\sqrt{1-\sin^2 A}$

$= \sin A \times \sin A + \cos A \times \cos A$

$= \sin^2 A + \cos^2 A$

$= 1$

$= R.H.S$ proved

xii) $\cos A.\sqrt{1+\tan^2 A} + \sin A\sqrt{1+\cot^2 A} = 2$

→ Sol?

Here,

$$L.HS = \cos A.\sqrt{1+\tan^2 A} + \sin A\sqrt{1+\cot^2 A}$$

$$= \cos A \times \sec A + \sin A \times \operatorname{cosec} A$$

$$= \cancel{\cos A} \times \frac{1}{\cancel{\cos A}} + \cancel{\sin A} \times \frac{1}{\cancel{\sin A}}$$

$$= 1+1$$

$$= 2$$

= Proved

(xiii) $\sin\alpha \cdot \cos\alpha(\tan\alpha + \cot\alpha) = 1$

→ Soln

Here,

$$L.HS = \sin\alpha \times \cos\alpha(\tan\alpha + \cot\alpha)$$

$$= \sin\alpha \times \cos\alpha\left(\frac{\sin\alpha}{\cos\alpha} + \frac{\cos\alpha}{\sin\alpha}\right)$$

$$= \sin\alpha \times \cos\alpha\left(\frac{\sin^2\alpha + \cos^2\alpha}{\sin\alpha\cos\alpha}\right)$$

$$= \sin\alpha \times \cos\alpha \times \frac{1}{\sin\alpha\cos\alpha}$$

$$= \cancel{\sin\alpha\cos\alpha} \times \frac{1}{\cancel{\sin\alpha\cos\alpha}}$$

$$= 1$$ = Proved